General Extracted Pole Synthesis Technique with Applications to Low-Loss TE_{011} Mode Filters

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Abstract—**A novel synthesis technique is developed for two-port networks which possess finite real frequency transmission zeros. The low-pass prototype is synthesized in the form of a network with complex conjugate symmetry where the real frequency transmission zeros are extracted from both ends and realized by simple resonators separated by phase shifters. The remaining transmission zeros are realized by the central part of the filter in the form of a cross-coupled double array.**

This prototype is particularly suitable for designing waveguides bandpass filters and each real frequency transmission zero is independently tunable. Furthermore, in the case of the most complex transfer function with all possible types of transmission zeros, the realization requires only one type of coupling which is necessary in the important case of TE_{011} cylindrical mode cavity resonators.

The general synthesis technique is given and the process illustrated by a nontrivial example. Additionally, from the results of a computer program based upon the synthesis techniques, the important differences between the possible prototype forms for the same transfer function resulting from extracting the transmission zeros in different orders are cited.

I. Introduction

LOW PASS transfer functions for two-port resistively terminated networks may be synthesized in many ways. The classical cascade synthesis techniques enable the transmission zeros to be independently realized by separate interacting two-port sections and are therefore attractive from a practical viewpoint. However, when the transmission zeros are complex or on the real axis of the complex plane, as is the case with selective linear phase filters with monotonic stopbands, then a more useful prototype is the cross-coupled double array [1]. One of the main applications is in microwave bandpass filters.

In both cases, when j-axis transmission zeros (real frequency transmission zeros) occur in the complex frequency variable, then each zero at either side of the passband is not independently tunable by a single component in the network. Since the location of this type of transmission zero is very sensitive, from a practical viewpoint it would be desirable to extract them from either above or below the passband in an independent manner.

This was appreciated in the development of the "natural prototype" for the elliptic function filter. In that case explicit formulas for the element values resulted from the synthesis procedure [2]. In this paper, this technique is extended for arbitrary transfer functions where the resonant circuits for each j-axis transmission zero are separated by unity impedance phase shifters.

Since most transfer functions resulting from approximation theory for low-pass filters [2] may be realized by a symmetrical structure, this will enable the network to be synthesized with complex conjugate symmetry. Such a restriction is implied in this paper.

Initially, the different extraction cycles required in the synthesis procedure are developed in a general manner. The first is the extraction of unity impedance phase shifters from either end of the network with complex conjugate symmetry. This is followed by the extraction of single shunt resonators from either end to extract a complementary pair of j-axis transmission zeros from each side of the passband. This process is repeated until all of the finite j-axis transmission zeros have been extracted and the remaining network is realized by a double cross-coupled array which also has complex conjugate symmetry.

The entire synthesis procedure is illustrated by a nontrivial example which possesses pairs of transmission zeros at infinity on the j-axis and on the real axis. Additionally, using a computer program based on the synthesis procedure, an example of a sixth degree elliptic function prototype with four j-axis transmission zeros is investigated with the four possible forms resulting from extracting the transmission zeros in a different order.

For any transfer function the synthesis technique results in all coupling coefficients being positive. Thus it is ideal for low-loss TE_{011} mode cavity filters with purely inductive iris coupling. The prototype synthesis has been extended to cover this waveguide filter and a sixth degree filter has been designed and built at 19.6 GHz.

This filter is a sixth-order pseudoelliptic type providing very sharp cutoff slopes for rejection of the spread spectrum of the preceding nonlinear power amplifier. The six TE_{011} mode cavities were formed together with the rectangular guide coupling the j-axis transmission zero cavities by milling two solid blocks of aluminium each forming half of a cavity.

The final device required little tuning and the experimental results are given. These results agree very closely with theory and even though the cavities were unplated and not polished, the loss was <0.7 dB with an estimated power handling capability of several kilowatts.

Manuscript received January 9, 1980; revised April 15, 1980.

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II. General Synthesis Procedure

The synthesis procedure is developed in terms of the transfer matrix using the fact that the network can be synthesised with complex conjugate symmetry. At any stage in the synthesis process the transfer matrix will be of the form

$$\frac{1}{jF}\begin{bmatrix} A_1+jA_2 & B \\ C & A_1-jA_2 \end{bmatrix} \tag{1}$$

where A_1 is an odd polynomial in the complex frequency variable p and F, A_2, B, and C are even polynomials. F will contain as factors the remaining transmission zeros and due to reciprocity

$$A_1^2+A_2^2-BC=-F^2. \tag{2}$$

A unity impedance phase shifter of phase angle ψ_1 may be extracted from the output and the complex conjugate section, i.e., a phase shifter with phase angle $-\psi_1$, may also be extracted from the input. This will leave a remaining transfer matrix of the form

$$\frac{1}{jF}\begin{bmatrix} \cos\psi_1 & j\sin\psi_1 \\ j\sin\psi_1 & \cos\psi_1 \end{bmatrix}\cdot\begin{bmatrix} A_1+jA_2 & B \\ C & A_1-jA_2 \end{bmatrix}\cdot\begin{bmatrix} \cos\psi_1 & -j\sin\psi_1 \\ -j\sin\psi_1 & \cos\psi_1 \end{bmatrix}$$

$$=\frac{1}{jF}\begin{bmatrix} A_1+j(A_2\cos 2\psi_1+\sin 2\psi_1(C-B)/2) & B\cos^2\psi_1+C\sin^2\psi_1+A_2\sin 2\psi_1 \\ C\cos^2\psi_1+B\sin^2\psi_1-A_2\sin 2\psi_1 & A_1-j(A_2\cos 2\psi_1+\sin 2\psi_1(C-B)/2) \end{bmatrix}. \tag{3}$$

If there is a pair of j-axis transmission zeros at $p=\pm j\omega$, then in order to extract them with shunt resonators we must first make the new B parameter in (3) possess a factor $(p^2+\omega_1^2)$. Hence

$$B\cos^2\psi_1+C\sin^2\psi_1+A_2\sin 2\psi_1\big|_{p=\pm j\omega_1}=0$$

$$t_1=\tan\psi_1=\left.\frac{-A_2\pm\sqrt{A_2^2-BC}}{C}\right|_{p=\pm j\omega_1} \tag{4}$$

which from (2) produces one solution

$$t_1=\left.\frac{-A_2+jA_1}{C}\right|_{p=j\omega_1}=\left.\frac{B}{-A_2-jA_1}\right|_{p=j\omega_1}. \tag{5}$$

This solution results in the new A parameter possessing a factor $(p+j\omega_1)$ and the new D parameter a factor $(p-j\omega_1)$. Thus the overall transfer matrix may be expressed as

$$T'=\frac{1}{jF}\begin{bmatrix} j(p+j\omega_1)(A_1'+jA_2') & B'(p^2+\omega_1^2) \\ C' & -j(p-j\omega_1)(A_1'-jA_2') \end{bmatrix} \tag{6}$$

where

$$C'=C\cos^2\psi_1+B\sin^2\psi_1-A_2\sin 2\psi_1$$

$$B'=\frac{B\cos^2\psi_1+C\sin^2\psi_1+A_2\sin 2\psi_1}{(p^2+\omega_1^2)}$$

$$A_1'+jA_2'=\frac{A_2\cos 2\psi_1+\sin 2\psi_1(C-B)/2-jA_1}{(p+j\omega_1)}. \tag{7}$$

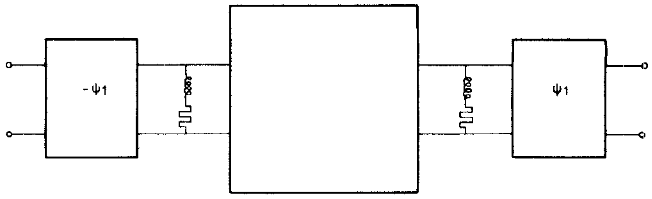

Fig. 1. A complete cycle for extracting j-axis transmission zeros.

We may now extract resonators of admittance

$$\frac{b_1}{p+j\omega_1}\ \text{and}\ \frac{b_1}{p-j\omega_1} \tag{8}$$

from the input and output, respectively, to give

$$\begin{bmatrix} 1 & 0 \\ \dfrac{-b_1}{p+j\omega_1} & 1 \end{bmatrix}\begin{bmatrix} T' \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \dfrac{-b_1}{p-j\omega_1} & 1 \end{bmatrix}$$

$$=\frac{1}{jF''}\begin{bmatrix} A_1''+jA_2'' & B'' \\ C'' & A_1''-jA_2'' \end{bmatrix} \tag{9}$$

where

$$B''=B'\quad F''=\frac{F}{(p^2+\omega_1^2)}$$

$$C''=\frac{C'+b_1^2B'+2b_1A_2'}{(p^2+\omega_1^2)}$$

$$A_1''+jA_2''=\frac{-A_2'-b_1B'+jA_1'}{p-j\omega_1} \tag{10}$$

with

$$b_1=\left.\frac{A_2'+jA_1'}{B'}\right|_{p=-j\omega_1}. \tag{11}$$

A complete cycle of the initial part of the synthesis procedure has now been completed and is illustrated in Fig. 1. This cycle may now be repeated until either all or the desired number of j-axis transmission zeros have been extracted in this manner.

The remaining network is to be synthesized as a cross-coupled double array. Assuming that there is at least a double ordered transmission zero at infinity, the first step is to extract unity impedance phase shifters to allow this pair of zeros at infinity to be extracted by shunt capacitors. The value of the phase shifter must be such that the degree of the remaining B parameter is two lower than the degree of the remaining C parameter. This can be obtained from (3) or, since the original network was real, the value of the phase is simply minus the sum of the phases of the phase shifters already extracted.

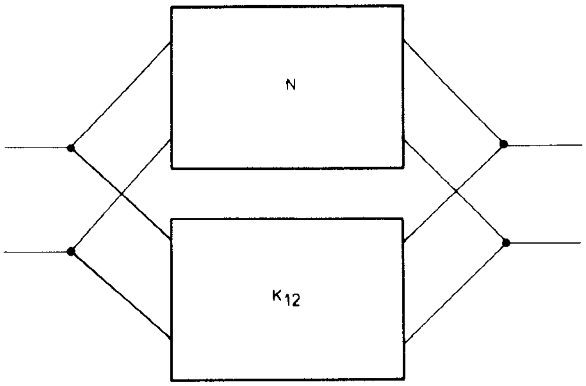

Fig. 2. Parallel extraction of an inverter in the cross-coupled array synthesis.

The transfer matrix will now be in the form given in (1) where B is two degrees lower than C. Extracting admittances C_1p+jB_1 and C_1p-jB_1 from the input and output, respectively, yields

$$\begin{bmatrix} 1 & 0 \\ -C_1p-jB_1 & 1 \end{bmatrix} \frac{1}{jF} \begin{bmatrix} A_1+jA_2 & B \\ C & A_1-jA_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -C_1p+jB_1 & 1 \end{bmatrix} = \frac{1}{jF} \begin{bmatrix} A_1'+jA_2' & B' \\ C' & A_1'-jA_2' \end{bmatrix} \quad (12)$$

where

$$\begin{aligned} A_1' &= A_1 - C_1pB \\ A_2' &= A_2 + B_1B \\ B' &= B \\ C' &= C - 2A_1C_1p + 2A_2B_1 + C_1^2p^2B + B_1^2B \end{aligned} \quad (13)$$

with

$$C_1 = \left.\frac{A_1}{pB}\right|_{p=\infty} \quad B_1 = \left.\frac{-A_2}{B}\right|_{p=\infty}. \quad (14)$$

To continue the cross-coupled array synthesis, an impedance inverter of characteristic admittance K_{12} must be extracted in parallel with the network such that the remaining two-port possesses a double ordered transmission zero at infinity. This is illustrated in Fig. 2 and the remaining two-port transfer matrix is

$$\frac{1}{j(F-K_{12}B')} \begin{bmatrix} A_1'+jA_2' & B' \\ C'-2FK_{12}+K_{12}^2B' & A_1'-jA_2' \end{bmatrix} \quad (15)$$

with

$$K_{12} = \left.\frac{F}{B'}\right|_{p=\infty}. \quad (16)$$

To complete a basic cycle, unity impedance, impedance inverters are extracted from both ends to leave the transfer matrix

$$\frac{1}{jF''} \begin{bmatrix} A_1''+jA_2'' & B'' \\ C'' & A_1''-jA_2'' \end{bmatrix} \quad (17)$$

where

$$\begin{aligned} F'' &= -(F-K_{12}B') \\ A_1'' &= A_1' \quad A_2'' = -A_2' \\ C'' &= B' \quad B'' = C' - 2FK_{12} + K_{12}^2B' \end{aligned} \quad (18)$$

with the complete cycle being illustrated in Fig. 3. This cycle is then repeated until the network has been completely synthesized.

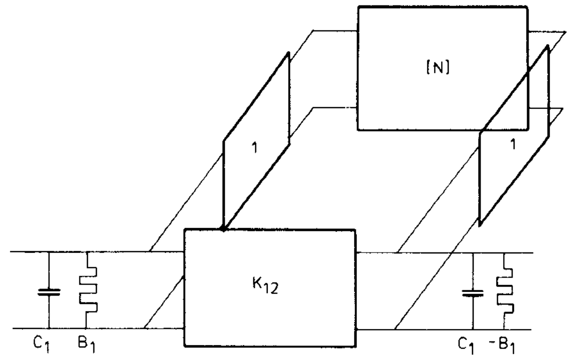

Fig. 3. A complete cycle in the cross-coupled array synthesis.

III. Example of the Synthesis Procedure

An example of a sixth degree network synthesized in a symmetrical cross-coupled array form [1] is, from the input

Capacitance	Coupling Admittance
$C_1 = 1.000367$	$K_1 = -0.078796$
$C_2 = 1.43354$	$K_2 = -0.000237$
$C_3 = 1.938932$	$K_3 = 1.184116$

(19)

This example possesses a pair of transmission zeros at infinity, a pair on the j axis at $p = \pm j1.414$, and a pair on the real axis at $p = \pm 0.953076$. Furthermore, it has an equiripple passband amplitude characteristic with a minimum passband return loss of 20 dB for $|\omega| \leq 1$ and an equiripple group delay over 60 percent of the passband. Also, a stopband level of 28 dB is maintained for $\omega > 1.35$.

The first step is to construct the even mode input admittance from which the transfer matrix may then be obtained. The even mode admittance is

$$Y_e = C_1p + jK_1 + \cfrac{1}{C_2p + jK_2 + \cfrac{1}{C_3p + jK_3}}$$

$$= \frac{2.78056p^3 + jp^2 1.47862 + p3.0733 + j1.1053}{2.77954p^2 + jp1.69702 + 1.00028} \quad (20)$$

The odd mode admittance is $Y_o = Y_e^*$ and since the transfer matrix is

$$\frac{1}{jF} \begin{bmatrix} A & B \\ C & A \end{bmatrix} = \frac{1}{Y_e - Y_o} \begin{bmatrix} Y_e + Y_o & 2 \\ 2Y_eY_o & Y_e + Y_o \end{bmatrix} \quad (21)$$

we have

$$\begin{aligned} F &= 1.10561 - 0.664187p^2 - 0.608764p^4 \\ F &= \left(p^2 + (1.414)^2\right)(0.552973 - 0.608764p^2) \\ A &= 4.94987p + 13.8329p^3 + 7.72866p^5 \\ B &= 1.00056 + 8.4405p^2 + 7.72582p^4 \\ C &= 1.22168 + 12.7138p^2 + 19.2773p^4 + 7.7315p^6. \end{aligned} \quad (22)$$

Choosing the phase shifter according to (4) gives

$$\psi_1 = -52.3531^0 \quad (23)$$

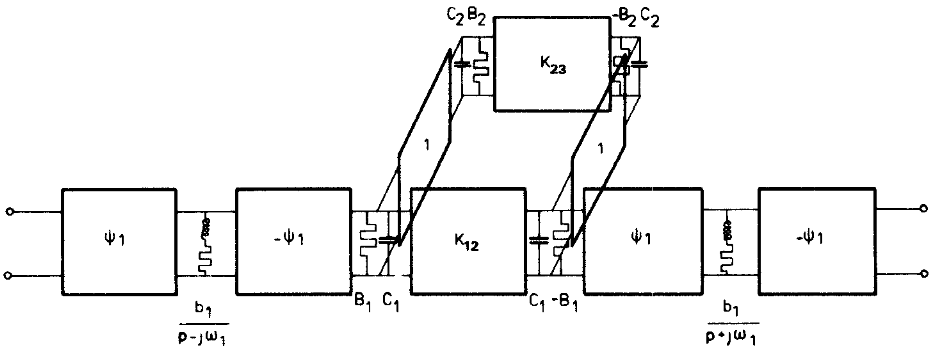

Fig. 4. Final network resulting from the sixth degree linear phase prototype with real frequency transmission zeros at $\omega = \pm 1.414$

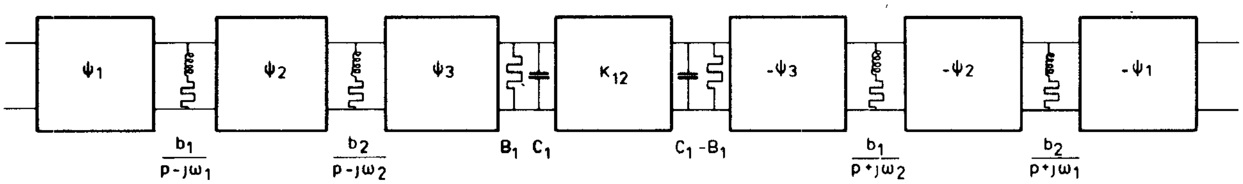

Fig. 5. The network resulting from the synthesis of a sixth degree elliptic function filter.

and the transfer matrix (3) becomes

$$\frac{1}{jF}\begin{bmatrix} A_1+jA_2 & B \\ C & A_1-jA_2 \end{bmatrix} \tag{24}$$

with

$$A_1 = 4.94987p + 13.8329p^3 + 7.72866p^5$$
$$A_2 = 0.106939 + 2.06665p^2 + 5.58652p^4 + 3.73911p^6$$
$$C = 1.08306 + 10.0347p^2 + 12.0353p^4 + 2.88438p^6$$
$$B = 1.13919 + 11.1196p^2 + 14.9678p^4 + 4.84712p^6. \tag{25}$$

Factorizing the matrix in the form described by (6) and (7) gives

$$C' = 1.08306 + 10.0347p^2 + 12.0353p^4 + 2.88438p^6$$
$$B' = 0.569767 + 5.27649p^2 + 4.84712p^4$$
$$A_1' + jA_2' = 0.0756286j + 3.5541p - 1.05194jp^2 + 9.03889p^3 - 2.44156jp^4 + 3.73911p^5$$
$$F = (p^2 + 1.999396)(0.552973 - 0.608764p^2). \tag{26}$$

From (11) the residue for the extracted resonant section is

$$b_1 = 0.871402 \tag{27}$$

resulting in the remaining transfer matrix described by (10) being

$$B'' = 0.569767 + 5.27649p^2 + 4.84712p^4$$
$$F'' = 0.552973 - 0.608764p^2$$
$$C'' = 0.824005 + 5.69375p^2 + 2.88438p^4$$
$$A_1'' + jA_2'' = 0.404614 - 2.22736jp + 4.083p^2 - 3.50487jp^3 + 3.73911p^4. \tag{28}$$

The value of the phase shifter which must now be extracted to allow shunt capacitors to be subsequently extracted is $-\psi_1$. Extracting this phase shifter using (3) gives

$$F = 0.552973 - 0.608764p^2$$
$$C = 1.05597 + 9.38141p^2 + 7.7315p^4$$
$$B = 0.337797 + 1.58884p^2$$
$$A_1 = 2.22736p + 3.50487p^3$$
$$-A_2 = 0.225671 + 1.23832p^2. \tag{29}$$

From (14) we have

$$C_1 = 2.20593 \quad B_1 = 0.779383 \tag{30}$$

and the remaining transfer matrix (12) is defined by the parameters in (13) as

$$A_1' = 1.4822p$$
$$A_2' = 0.0376023$$
$$B' = 0.337797 + 1.58884p^2$$
$$C' = 0.909396 + 0.233249p^2$$
$$F = 0.552973 - 0.608764p^2. \tag{31}$$

From (16) the admittance inverter which must be extracted in parallel from the network is given by

$$K_{12} = -0.38315 \tag{32}$$

and extracting a further pair of unity impedance, impedance inverters in cascade from the remaining network gives the transfer matrix described in (17) and (18) as

$$F'' = -0.6824$$
$$A_1'' = 1.4822p$$
$$A_2'' = -0.0376023$$
$$C'' = 0.337797 + 1.58884p^2$$
$$B'' = 1.38273. \tag{33}$$

Returning to the cycle described in (12)–(16) gives the final elements as

$$C_2=1.07194 \quad B_2=0.0271943 \quad K_{23}=-0.493517. \quad (34)$$

The final network is shown in Fig. 4 with the sign of K_{12} and K_{23} changed, introducing 180° phase shift in the transfer function.

IV. Example on 6th Degree Elliptic Function Prototype

For an elliptic function filter with a passband return loss of 26 dB with a minimum stopband level of 37.5 dB for $\omega > 1.3345$, the transfer function possesses two transmission zeros at infinity and the remaining four at

$$p=\pm j1.367782=\pm j\omega_1$$
$$p=\pm j1.774078=\pm j\omega_2. \quad (35)$$

The pole locations are

$$p=-0.0920031\pm j1.1043378$$
$$p=-0.3681784\pm j.9752846$$
$$p=-0.7677068\pm j.4524555. \quad (36)$$

The even mode input admittance may readily be obtained from using the fact that the zeros of $1+Y_e$ are the alternate pole locations of the transfer function with interlacing imaginary parts. Thus

$$\text{Num}(1+Y_e)=(p+.0920031+j1.1043378)(p+.3681784-j.9752846)(p+.7677068+j.4524555) \quad (37)$$

from which Y_e may be obtained and the network synthesized in a symmetrical manner to give

Capacitance	Coupling Admittance
0.814406	0.059612
1.419191	−0.432731
2.136197	1.737211

(38)

Using the pole extraction synthesis technique we will have the network shown in Fig. 5. Since the four finite transmission zeros may be extracted in any order there are four possible network realizations. Using a computer program to synthesize the networks we have the resulting element values summarized in the following table.

Case 1

Phase Shift°	Trans. Zero	Residue
−68.17	1.3678	0.6120
−76.20	−1.7741	1.6156
144.37		
Cap.	Susc.	Adm. Inv.
6.5704	−0.2815	3.8800

Case 2

Phase Shift°	Trans. Zero	Residue
−68.17	1.3678	0.6120
48.02	1.7741	4.7971
20.15		
Cap.	Susc.	Adm. Inv.
7.6218	1.9501	5.2212

Case 3

Phase Shift°	Trans. Zero	Residue
−45.60	1.7741	1.6622
30.69	1.3678	3.7469
14.91		
	Cap. Susc.	Adm. Inv.
7.6218	1.9501	5.2212

Case 4

Phase Shift°	Trans. Zero	Residue
−45.60	1.7741	1.6622
84.93	−1.3678	0.5654
−39.33		
Cap.	Susc.	Adm. Inv.
6.5704	0.2815	3.8800

For most physical realizations based upon this type of prototype, it is desirable to have the residues as low as possible. Since any integer value of 180 may be added to any of the phase shifters, all phase shifts can be made positive. Thus, accounting for these facts, cases 1 and 4 produce more desirable resonator impedance values and similarly phase shifts between resonators. Therefore this suggests that it is probably more desirable to alternate the transmission zeros from either side of the passband on one side of the network when synthesizing the network.

We may now apply this general synthesis technique to waveguide filters.

V. Waveguide Realization

A low-pass prototype for an extracted pole filter is shown in Fig. 4, and the aim now is to convert this type of network into one which is suitable for realization in waveguide (Fig. 6). The process will be dealt with in three stages, firstly the synthesis of the cross-coupled body of the filter, then the synthesis of the pole cavities and finally the phase lengths between the pole cavities and the main body of the filter.

A. Synthesis of the Cross-Coupled Body

The synthesis of the body follows very similar lines to the synthesis of the synchronously tuned cross-coupled waveguide filter from the general low-pass cross-coupled array as described in [1] and [5]. The derivations will not be repeated here, but the final design formulas will be quoted:

$$B'_r=K_{r,r+1}/\alpha C_r$$
$$B'_{r-1,r}=\alpha\sqrt{C_rC_{r-1}}-1/\alpha\sqrt{C_rC_{r-1}}$$
$$\phi_{Lr}=\phi_{Ur}=\frac{\pi}{2}-\frac{1}{2}\left(\cot^{-1}\left(\frac{B'_{r-1,r}}{2}\right)+\sin^{-1}(B'_r)\right)$$
$$\theta_{Lr}=\theta_{Ur}=\frac{\pi}{2}-\frac{1}{2}\left(\cot^{-1}\left(\frac{B'_{r,r+1}}{2}\right)+\sin^{-1}(B'_r)\right)$$
$$C_0=\frac{1}{\alpha} \qquad C_{n+1}=\infty, \qquad r=1,2,\cdots,n$$
$$\alpha=\omega'(\lambda_{g1}+\lambda_{g2})/\pi(\lambda_{g1}-\lambda_{g2}) \quad (39)$$

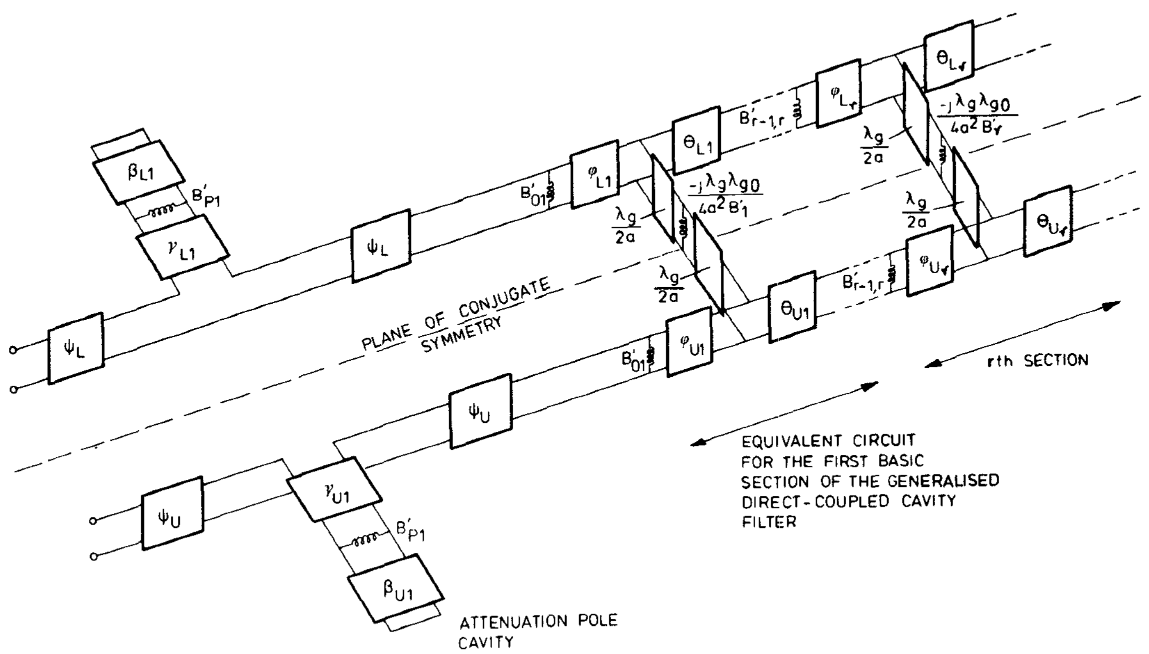

Fig. 6. Equivalent circuit for an extracted-pole filter in rectangular waveguide. λ_g = guide wavelength. λ_{go} = normalizing guide wavelength. a = rectangular waveguide broad dimension.

where λ_{g1}, λ_{g2} are the waveguide wavelengths at the lower and upper band edge frequencies, respectively, ω' is the low-pass prototype cutoff frequency, and the rest of the symbols have meanings as depicted in Figs. 4, 5, or 6.

As with the synchronously tuned filter, synthesis starts with the insertion of an open-circuit plane along the line of conjugate symmetry of Fig. 4 (i.e., through the center of the cross-coupling immittance inverters $K_{12}, K_{23}, \cdots$), whereupon an even mode network for the body of the filter will result as shown in Fig. 7.

The shunt frequency-invariant susceptances jK_{12}, jK_{23} are the equivalent elements for the cross-coupling inverters K_{12}, K_{23} shown in Fig. 4, of the same susceptance values as the characteristic admittance of the corresponding cross coupling inverters. The even-mode network for the other arm of the cross-coupled network is the same except B_r is replaced by $-B_r$. For the odd mode, K is replaced by $-K$.

By comparing this prototype even-mode network with the one used initially to synthesize the symmetric synchronously tuned waveguide filter in [1], it is clear that the only difference is the inclusion of the frequency-invariant susceptances B_r in parallel with the other two susceptances at each mode. Unlike the K susceptances the B susceptances do not change sign between the even and odd modes of the prototype network, and so may be realized by small lengths of transmission line to be added in at each node. The formulas for the phase lengths of the two halves of each cavity therefore become modified

$$\phi_{Ur}=\frac{\pi}{2}-\frac{1}{2}\left(\cot^{-1}\left(\frac{B'_{r-1,r}}{2}\right)+\sin^{-1}(B'_r+B'_{cr})\right)$$

$$\theta_{Ur}=\frac{\pi}{2}-\frac{1}{2}\left(\cot^{-1}\left(\frac{B'_{r,r+1}}{2}\right)+\sin^{-1}(B'_r+B'_{cr})\right)$$

$$\phi_{Lr}=\frac{\pi}{2}-\frac{1}{2}\left(\cot^{-1}\left(\frac{B'_{r-1,r}}{2}\right)+\sin^{-1}(B'_r-B'_{cr})\right)$$

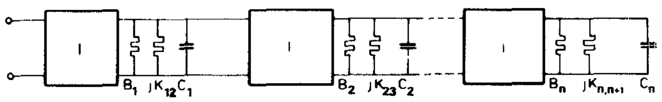

Fig. 7. Even-mode network for the complex-conjugate-symmetric array.

$$\theta_{Lr}=\frac{\pi}{2}-\frac{1}{2}\left(\cot^{-1}\left(\frac{B'_{r,r+1}}{2}\right)+\sin^{-1}(B'_r-B'_{cr})\right) \quad (40)$$

where

$$B'_{cr}=B_r/\alpha C_r.$$

Thus the resonators of the two halves of the cross-coupled network become complex-conjugately tuned and slightly different in length. In practice the length difference between corresponding resonators in the two halves of the network is small and may be totally taken up by tuning screws.

The shunt susceptances $B'_{r,r+1}$ in the main lines and the cross coupling shunt susceptances B'_r are realized by inductive irises, as usual.

B. Synthesis of the Pole Cavities

The pole cavity pairs may be synthesized by assuming that each pair forms the two arms of a single-section complex-conjugate cross-coupled array, but with no cross coupling. Thus the formulas (39) and (40) apply with $n=1$ and $B'_r=0$

$$B'_{p1}=\sqrt{\alpha C_{p1}}-1/\sqrt{\alpha C_{p1}}$$

$$B'_{cp1}=\frac{B_{p1}}{\alpha C_{p1}} \qquad B'_r=0$$

$$\beta_{U1}=\phi_{Up1}+\theta_{Up1}=\pi-\frac{1}{2}\cot^{-1}\frac{(B'_{p1})}{2}-\sin^{-1}(B'_{cp1})$$

$$\beta_{L1}=\phi_{Lp1}+\theta_{Lp1}=\pi-\frac{1}{2}\cot^{-1}\frac{(B'_{p1})}{2}+\sin^{-1}(B'_{cp1}). \quad (41)$$

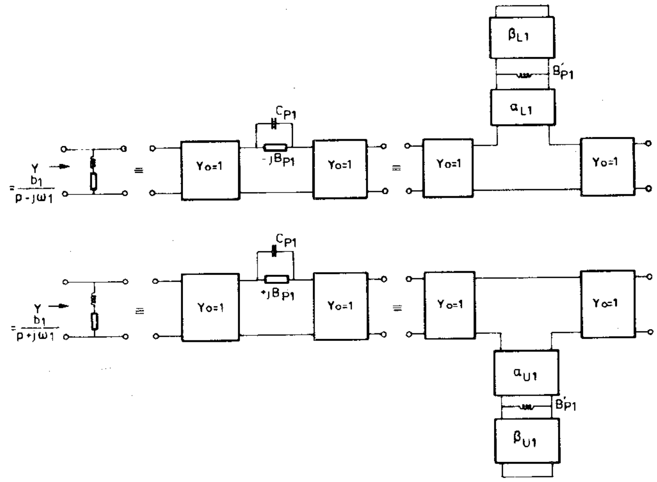

Fig. 8. Synthesis of pole cavities. (a) Low-pass equivalent circuits. (b) Dual circuits. (c) Waveguide equivalent circuits.

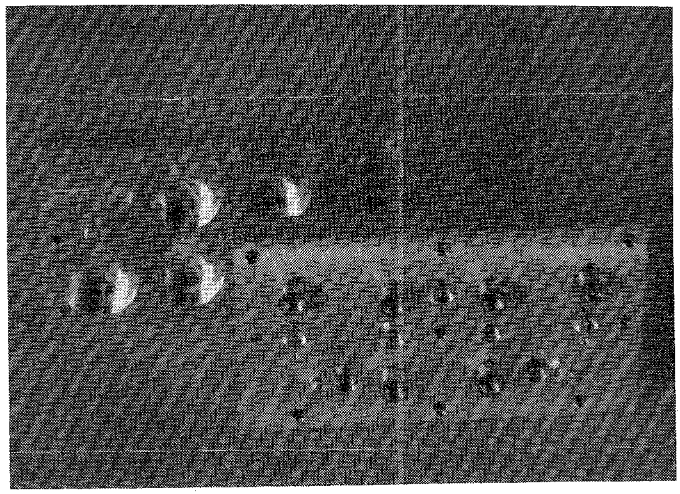

Fig. 9. Sixth degree pseudoelliptic extracted-pole filter.

Each pole is extracted as an inductor in series with a frequency invariant reactive component, the series pair in shunt across the transmission line (Fig. 4 and Fig. 8(a)). Fig. 8(b) gives the dual of this shunt circuit, whereby equating slope reactances, slope susceptances and resonant frequencies it can be determined that $C_{p1}=1/b_1$, and $B_{p1}=\pm\omega_1/b_1$; b_1 is the residue of the first extracted pole and ω_1 its corresponding low-pass prototype resonant frequency. As with the body of the filter the shunt reactances are realized as irises, and the phase lengths β_{L1} and β_{U1} as short-circuited lengths of waveguide about half a wavelength long, according to (41).

To compensate for the short negative lengths of transmission line associated with the iris susceptances, lengths of waveguide γ_U, γ_L approximately $\lambda_g/2$ in length, are included between the resonator irises and the main waveguide. If the lengths are normalized at the resonant frequencies of the cavities $\gamma_{U1}=\beta_{U1}$ and $\gamma_{L1}=\beta_U$.

C. Phase Lengths Between Filter Body and Pole Cavities

The unity admittance lengths of waveguide between the filter body and the complex conjugate pole cavities have to accommodate three factors: a) the phase lengths ψ (see Fig. 4); b) The short negative length of transmission line associated with the input and output susceptances of the body of the filter; and c) the admittance inverter associated with the equivalent circuit of the pole cavity (Fig. 8b and 8c). Therefore the prototype phase length between the body of the filter and the first cavity is modified

$$\psi_p \rightarrow \psi_p - \frac{1}{2}\cot^{-1}\left(\frac{B'_{U1}}{2}\right) + \frac{\pi}{2} \pm k\pi, \qquad k=\text{integer}. \quad (42)$$

D. Phase Lengths Between Adjacent Pole Cavities

For the phase lengths between adjacent pole cavities, adjacent admittance inverters in effect cancel, and no account need be taken of the effect of the first body iris. The phase lengths need only be modified by an arbitrary number of half-wavelengths which should be added or subtracted for a convenient physical configuration

$$\psi_p \rightarrow \psi_p \pm k\pi. \quad (43)$$

Computer analysis of narrow bandwidth filters shows that the frequency dependence of these connecting lines do not have a significant effect upon the performance of the filter.

E. Laboratory Model

To meet a requirement for an output filter for a high speed QPSK millimeter-wave repeater, an extracted pole filter was constructed. The principal requirements were low loss, which necessitated the use of the high-Q TE_{011} mode cavity, a planar construction to enable heat to be efficiently transferred to a flat cooling plate, and steep cutoff slopes to prevent the spread spectrum of the nonlinear TWT from interfering with adjacent channels. To meet these specifications a sixth order single-pole-pair pseudoelliptic filter was constructed using cylindrical-cavity TE_{011} mode resonators. Center frequency was 19.628 GHz and bandwidth 82 MHz.

A photograph of the laboratory model is shown in Fig. 9, and the measured performances are shown in Fig. 10 (attenuation characteristic), Fig. 11 (group delay), Fig. 12 (return loss), and Fig. 13 (wide-band spurious search). Agreement with the predicted responses is close, and band-center loss is 0.7 dB corresponding to a Q of about 9000. This will probably improve with cavity polishing and plating. The diameter of a cavity is about 2 cm which is still mechanically easy to work with, and the iris size is such that power handling capability is estimated to be about 5 kW [3].

F. Single-Sided Derivatives

Eventually it is hoped to use this type of filter for a millimeter wave output diplexer for two adjacent channels. For this application a steep cutoff is needed on one side of each filter, while the other may be relatively gentle. Since it is possible to tune the pole cavities separately, either cavity may be effectively removed by tuning it well away from the center frequency. Small adjustments to the other irises (with screws) were then necessary to restore the return loss to >20 dB over the channel bandwidth. The measured characteristics are shown for the single-

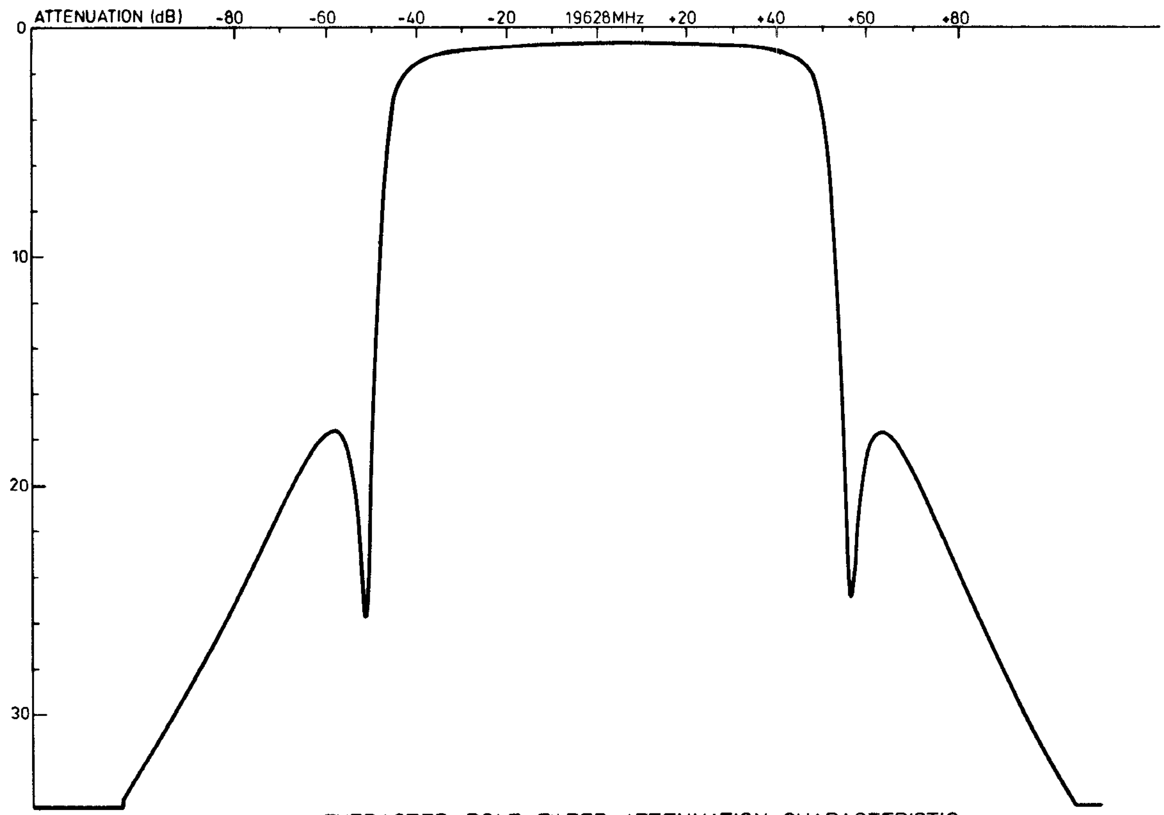

Fig. 10. Symmetric attenuation characteristic.

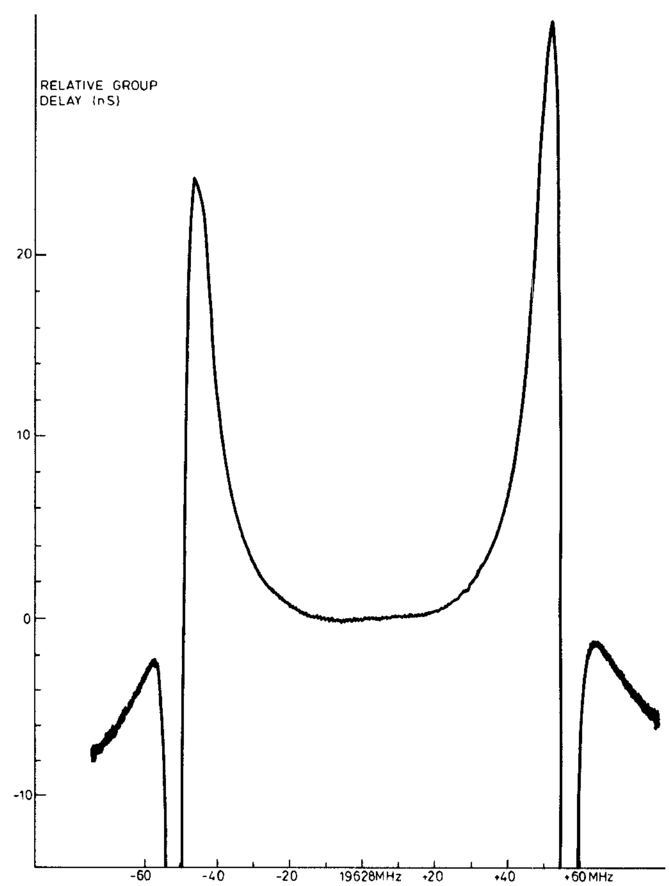

Fig. 11. Group delay, symmetric characteristic.

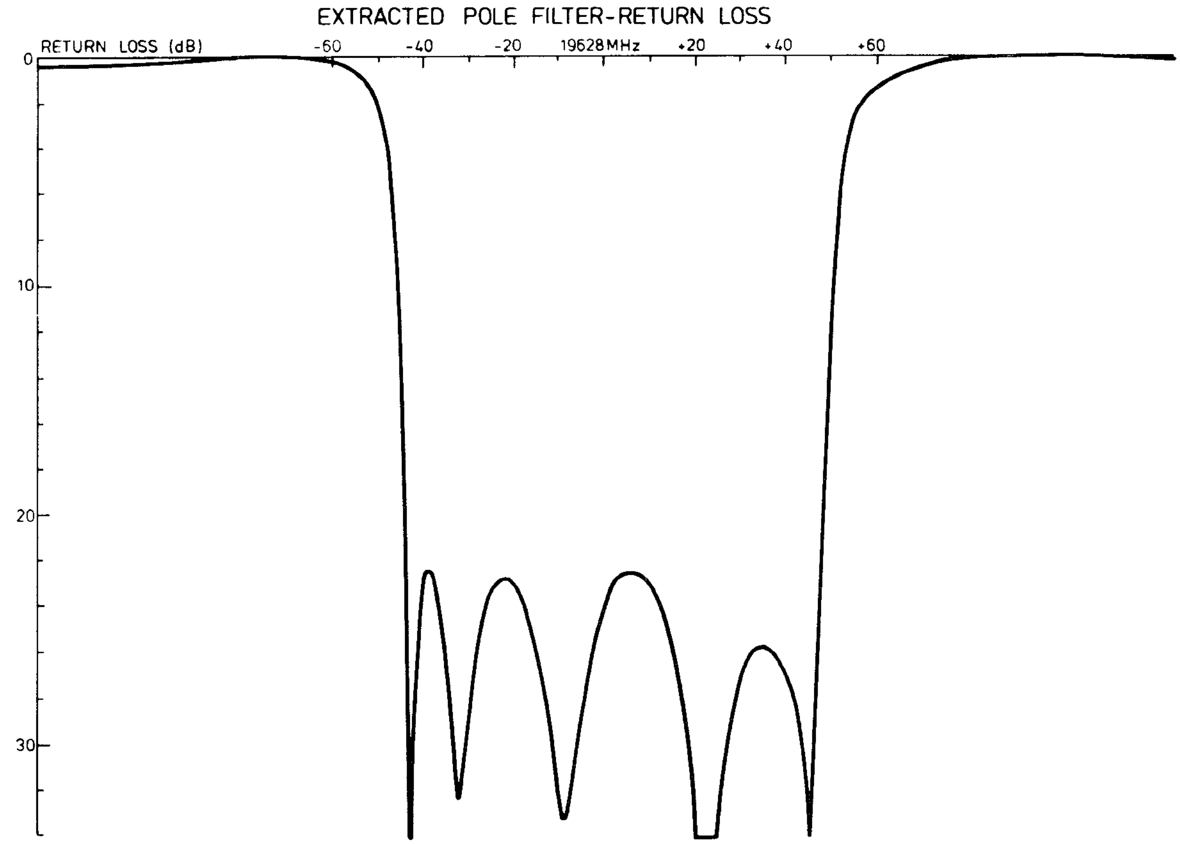

Fig. 12. Return loss, symmetric characteristic.

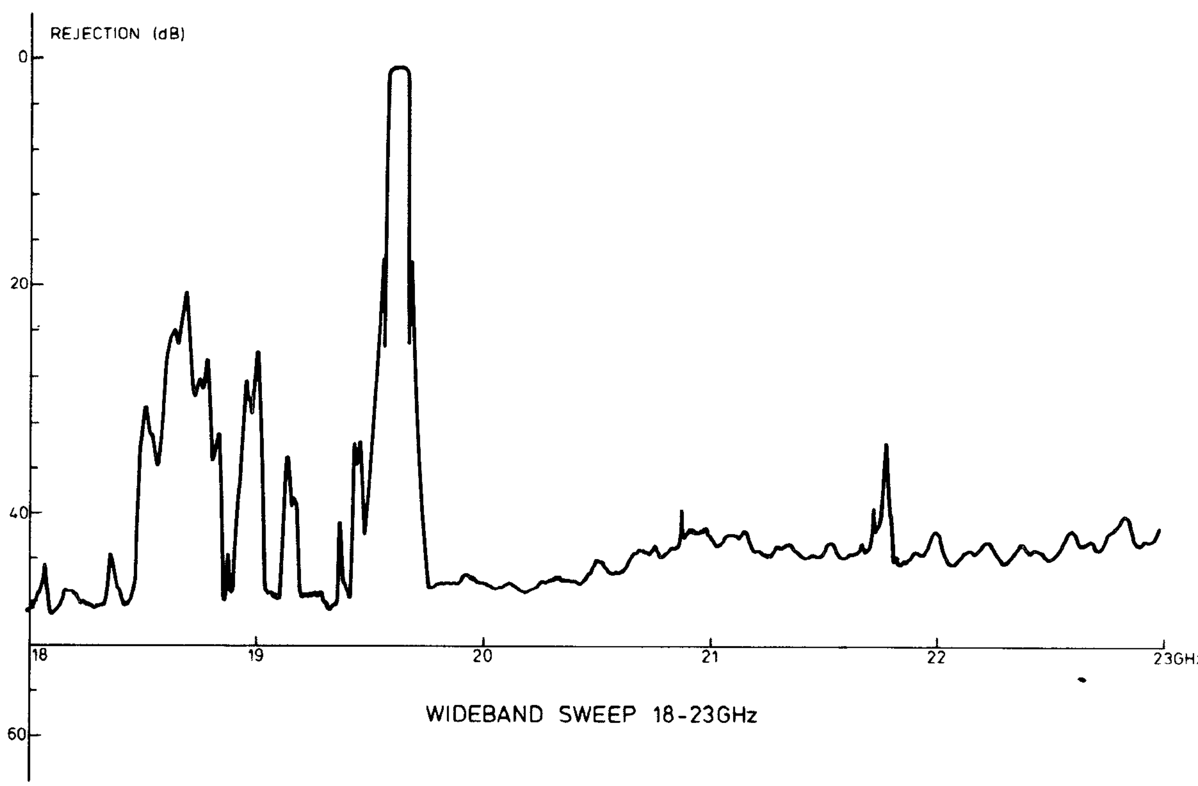

Fig. 13. Wide-band sweep, 18–23 GHz.

sided filter, pole on the lower side, in Fig. 14 (attenuation), Fig. 15 (return loss) and Fig. 16 (group delay), and the corresponding characteristics for the upper-pole single-sided filter in Figs. 17–19.

VI. Conclusion

A new synthesis procedure has been presented for the realization of the most general type of transfer function used in prototype filters. The pole extraction technique

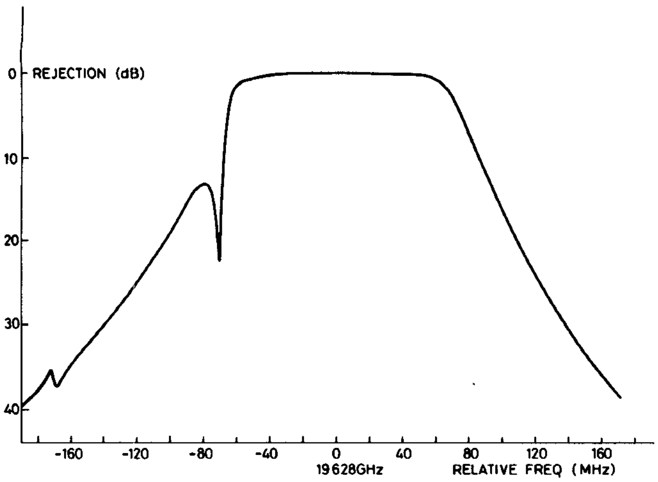

Fig. 14. Attenuation characteristic, pole on lower side.

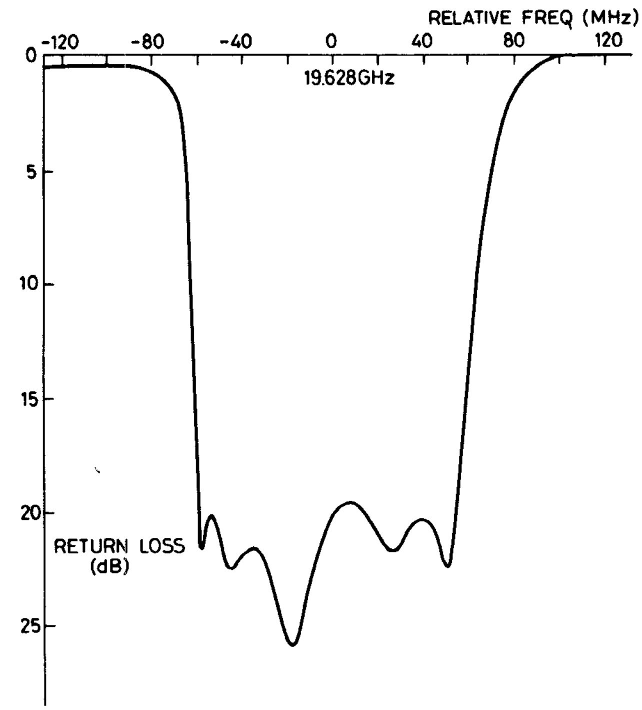

Fig. 15. Return loss, pole on lower side.

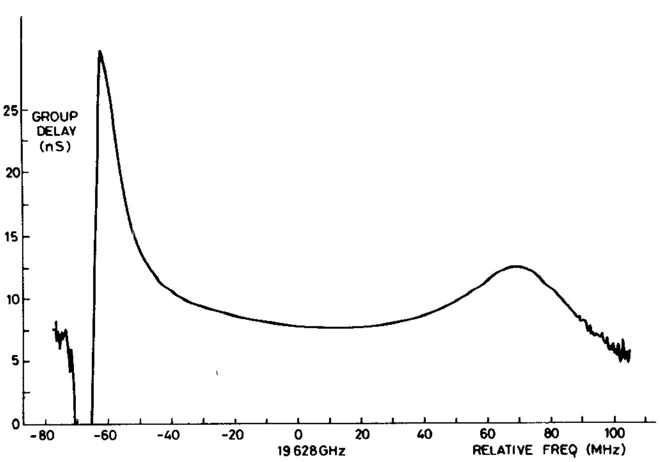

Fig. 16. Group delay, pole on lower side.

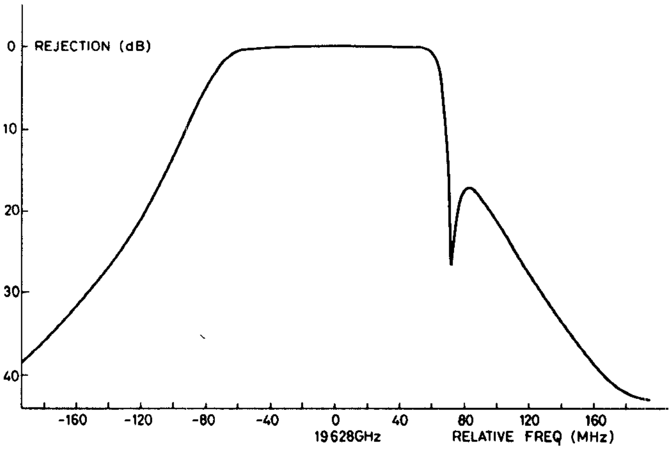

Fig. 17. Attenuation characteristic, pole on upper side.

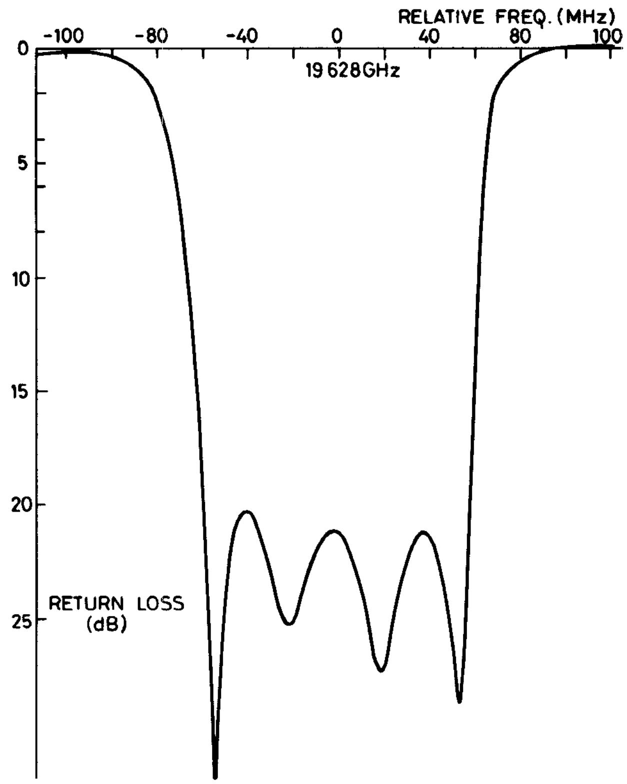

Fig. 18. Return loss, pole on upper side.

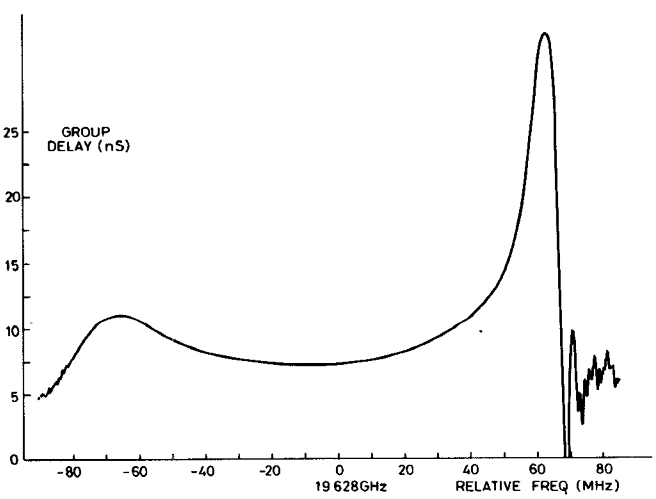

Fig. 19. Group delay, pole on upper side.

allows the real frequency transmission zeros to be extracted independently and realized by individual elements. This is important from a sensitivity view point. The remaining transmission zeros are realized in the center of the network with a cross-coupled array structure.

One of the most important results is that selective linear phase filters with finite real frequency transmission zeros may be realized with only positive couplings in the entire network. This is a necessary condition for many types of physical realizations particularly in waveguide structures when cavities only supporting a single mode of operation are used.

From this prototype, many narrow-band bandpass structures may be designed using the reactance slope parameter technique. At microwave frequencies, the phase shifters are realized by lengths of line or waveguide and the remaining elements by iris coupled cavities. To demonstrate the feasibility of the method a sixth degree pseudoelliptic bandpass filter has been constructed using low-loss TE_{011} mode cavities.

Having established the procedure for realizing combined pseudoelliptic and nonminimum phase (self-equalized) characteristics with the low-loss TE_{011} mode cavity, the other advantages of using this mode should be mentioned. The dimensions are comparatively large which mitigates multipactor effect in space, ensures high power handling and renders construction easy at millimeter wave frequencies up to about 40 GHz. The planar construction enables the device to be mounted on a flat cooling plate for efficient transfer of dissipated heat. The independent tuning for the pole cavities simplifies the tuning procedure for the filter.

Perhaps the most important application for this type of filter realization will be in the high-power low-distortion output multiplexion of contiguous or near-contiguous channels. A procedure is currently under development to match-multiplex extracted pole filters onto a common waveguide manifold.

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Short Papers

On the Design of Temperature Stabilized Delay Lines

PEITRO DE SANTIS

Abstract—**Recently published design formulas for delay lines with transmission phase temperature stabilization are shown to be approximate. Their validity range is assessed. New exact formulas of broader validity are presented.**

With the advent of low-loss microwave ceramic materials featuring negative dielectric constant temperature coefficients as well as high dielectric constants, a new class of temperature stabilized delay lines has recently become feasible. In [1] design criteria have been presented for temperature stabilized transmission line delay lines (TLDL) made up of barium tetratitanate and sapphire MIC's. These devices, to be used in satellite born regenerators for PSK data transmission, must provide a fixed group delay and a temperature stable transmission phase. To meet these requirements the lengths l_A and l_B of the $BaTi_4O_9$ and Al_2O_3 TLDL's (hereafter referred to as partial delay lines (A) and (B)) are calculated by solving the system

$$\frac{l_A}{v_{gA}} + \frac{l_B}{v_{gB}} = \tau \tag{1}$$

$$\left(\frac{\Delta\phi_A}{\Delta T} + \frac{\Delta\phi_B}{\Delta T}\right)(\phi_A + \phi_B)^{-1} = \alpha_\phi = 0 \tag{2}$$

where $V_{gA,B}$ and $\phi_{A,B}$ are the group velocities and the transmission phases associated with the two partial delay lines. Furthermore τ is the total group delay and α_ϕ the total transmission phase temperature coefficient over the temperature range ΔT.

In [1], (2) was approximated by

$$\frac{\alpha_A l_A}{v_{gA}} + \frac{\alpha_B l_B}{v_{gB}} = 0 \tag{3}$$

where $\alpha_{A,B}$ are the transmission phase temperature coefficients of the two partial delay lines.

Obviously the approximation is represented by the fact that phase velocities $v_{ph\,A,B}$ have been replaced by the corresponding group velocities $v_{gA,B}$. On the basis of that, one might think that

Manuscript received January 29, 1980; revised April 1, 1980. This paper was supported in part by ESTEC under Contract 3643/78/NL.

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0018-9480/80/0900-1028$00.75 © 1980 IEEE